On boundedness of the maximal operator in Morrey-type spaces

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Under discussion will be the boundedness of the classical Hardy-Littlewood maximal operator

$$(Mf)(x) = \sup_{r>0} \frac{1}{|B(x,r)|} \int_{B(x,r)} |f(y)|dy, \qquad x \in \mathbb{R}^n ,$$

where $B(x,r)$ is the ball centered at x of radius r in local and global Morrey-type spaces $LM_{p\theta,w(\cdot)}$, $GM_{p\theta,w(\cdot)}$ respectively, characterized by numerical parameters $0 < p, \theta \leq \infty$ and a functional parameter w (non-negative measurable function on $(0,\infty)$), namely the spaces of all functions $f \in L_p^{loc}(\mathbb{R}^n)$ with finite quasi-norms

$$\|f\|_{LM_{p\theta,w(\cdot)}} = \Big\| w(r) \, \|f\|_{L_p(B(0,r))} \Big\|_{L_\theta(0,\infty)} ,$$

$$\|f\|_{GM_{p\theta,w(\cdot)}} = \sup_{x\in\mathbb{R}^n} \|f(x+\cdot)\|_{LM_{p\theta,w(\cdot)}}$$

respectively.

This problem is reduced to the problem of boundedness of the so-called supremal operator from one weighted Lebesgue space to another one on the cone of non-negative non-increasing functions. This allows obtaining, for all admissible values of the numerical parameters $\alpha, p_1, p_2, \theta_1, \theta_2$, sufficient conditions on the functional parameters w_1 and w_2 ensuring the boundedness of M from $LM_{p_1\theta_1,w_1(\cdot)}$ to $LM_{p_2\theta_2,w_2(\cdot)}$ and from $GM_{p_1\theta_1,w_1(\cdot)}$ to $GM_{p_2\theta_2,w_2(\cdot)}$. Moreover, for a certain range of the numerical parameters, these sufficient conditions coincide with the necessary ones.

[1] Burenkov V. I., Guliyev H. V. Necessary and sufficient conditions for boundedness of the maximal operator in the local Morrey-type spaces. Dokl. Akad. Ross. Nauk. Matematika. **391**, no **5** (2003), 591–594 (Russian). English transl. in Russian Acad. Sci. Dokl. Math. **67** (2003).

[2] Burenkov V. I., Guliyev H. V. Necessary and sufficient conditions for boundedness of the maximal operator in the local Morrey-type spaces. Studia Mathematica, **163** **(2)** (2004), 157–176.

[3] Burenkov V. I., Gogatishvili A., Guliyev V. S., Mustafaev R., Boundedness of the fractional maximal operator in local Morrey-type spaces. Complex Analysis and Elliptic Equations **55** (2010), no. 8-10, 739-758.