Equivalent Boundedness of Marcinkiewicz Integrals on Non-Homogeneous Metric Measure Spaces

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Abstract

Let $(\mathcal{X}, d, \mu)$ be a metric measure space satisfying the upper doubling condition and the geometrically doubling conditions in the sense of T. Hytönen. We prove that the $L^p(\mu)$ boundedness with $p \in (1, \infty)$ of the Marcinkiewicz integral is equivalent to either of its boundedness from $L^1(\mu)$ into $L^{1,\infty}(\mu)$ or from the atomic Hardy space $H^1(\mu)$ into $L^1(\mu)$. Moreover, we show that if the Marcinkiewicz integral is bounded form $H^1(\mu)$ into $L^1(\mu)$, then it is also bounded from $L^\infty(\mu)$ into the space $\mathrm{RBLO}(\mu)$ (the regularized BLO), which is a proper subset of $\mathrm{RBMO}(\mu)$ (the regularized BMO) and, conversely, if the Marcinkiewicz integral is bounded from $L_b^\infty(\mu)$ (the set of all $L^\infty(\mu)$ functions with bounded support) into the space $\mathrm{RBMO}(\mu)$, then it is also bounded from the finite atomic Hardy space $H^{1,\infty}_{\mathrm{fin}}(\mu)$ into $L^1(\mu)$. These results essentially improve the known results even for non-doubling measures.