Generalized fractional integrals of variable order

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Let $\mathbb{R}^n$ be the n-dimensional Euclidean space. We denote by $B(x,r)$ the open ball centered at $x \in \mathbb{R}^n$ and of radius r, that is,

$$B(x,r) = \{y \in \mathbb{R}^n : |y - x| < r\}.$$

For a measurable set $G \subset \mathbb{R}^n$, we denote by $|G|$ and χ_G the Lebesgue measure of G and the characteristic function of G, respectively.

For a function $f \in L^1_{\mathrm{loc}}(\mathbb{R}^n)$ and a ball B, let

$$f_B = \fint_B f = \fint_B f(y)\,dy = \frac{1}{|B|}\int_B f(y)\,dy.$$

In this talk we consider generalized Morrey spaces $L^{(p,\varphi)}(\mathbb{R}^n)$ with variable exponent $p : \mathbb{R}^n \to [1,\infty)$ and variable growth condition $\varphi : \mathbb{R}^n \times (0,\infty) \to (0,\infty)$. For a ball $B = B(x,r)$ we write $\varphi(B) = \varphi(x,r)$. The definition is the following:

Definition 0.1. For $p : \mathbb{R}^n \to [1,\infty)$ and $\varphi : \mathbb{R}^n \times (0,\infty) \to (0,\infty)$, let $L^{(p,\varphi)}(\mathbb{R}^n)$ be the sets of all functions f such that the following functional is finite:

$$\|f\|_{L^{(p,\varphi)}(\mathbb{R}^n)} = \sup_B \|f\|_{p,\varphi,B},$$

where

$$\|f\|_{p,\varphi,B} = \inf\left\{\lambda > 0 : \frac{1}{\varphi(B)}\fint_B \left(\frac{|f(y)|}{\lambda}\right)^{p(y)} dy \le 1\right\}.$$

We show the boundedness of generalized fractional integral operators of variable order on $L^{(p,\varphi)}(\mathbb{R}^n)$.