CALDERÓN-VAILLANCOURT TYPE THEOREM FOR BILINEAR OPERATORS

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In this talk, we consider the boundedness of bilinear pseudo-differential operators. For $m \in \mathbb{R}$ and $0 \le \rho, \delta \le 1$, the bilinear Hörmander symbol class $BS^m_{\rho,\delta}$ consists of all $\sigma(x,\xi,\eta) \in C^\infty(\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n)$ such that

$$|\partial_x^\alpha \partial_\xi^\beta \partial_\eta^\gamma \sigma(x,\xi,\eta)| \le C_{\alpha,\beta,\gamma}(1+|\xi|+|\eta|)^{m+\delta|\alpha|-\rho(|\beta|+|\gamma|)}$$

for all multi-indices α, β, γ. For a symbol $\sigma \in BS^m_{\rho,\delta}$, the bilinear pseudo-differential operator T_σ is defined by

$$T_\sigma(f,g)(x) = \int_{\mathbb{R}^{2n}} e^{2\pi i x\cdot(\xi+\eta)} \sigma(x,\xi,\eta) \widehat{f}(\xi)\widehat{g}(\eta)\, d\xi d\eta, \qquad f,g \in \mathcal{S}(\mathbb{R}^n).$$

Let us consider the boundedness of T_σ from $L^2 \times L^2$ to L^1 for the sake of simplicity, since it is often understood that the $L^2 \to L^2$ boundedness of the linear case corresponds to the $L^2 \times L^2 \to L^1$ boundedness of the bilinear case. In the case $\rho = 1$, there does not seem to be a big difference between linear and bilinear pseudo-differential operators. In fact, Bényi-Maldonado-Naibo-Torres proved the boundedness of bilinear pseudo-differential operators with symbols in $BS^0_{1,\delta}$, $0 \le \delta < 1$, from $L^2 \times L^2$ to L^1. The case $\delta = 0$ is due to Coifman-Meyer. It should also be pointed out that bilinear pseudo-differential operators with symbols in $BS^0_{1,1}$ have Calderón-Zygmund kernels in the sense of Grafakos-Torres. In contrast to the case $\rho = 1$, there is a certain difference in the case $\rho < 1$. In fact, in the linear case, if a symbols $\sigma(x,\xi)$ satisfies

$$|\partial_x^\alpha \partial_\xi^\beta \sigma(x,\xi)| \le C_{\alpha,\beta}(1+|\xi|)^{\delta|\alpha|-\rho|\beta|},$$

where $0 \le \delta \le \rho < 1$, then the corresponding pseudo-differential operator is bounded on L^2. On the other hand, in the bilinear case, Bényi-Bernicot-Maldonado-Naibo-Torres proved that there exists a symbol $\sigma \in BS^0_{\rho,\delta}$ with $0 \le \delta \le \rho < 1$ such that T_σ is not bounded from $L^2 \times L^2$ to L^1. This means that if all bilinear pseudo-differential operators with symbols in $BS^m_{\rho,\delta}$ with $0 \le \delta \le \rho < 1$ are bounded from $L^2 \times L^2$ to L^1, then m must be negative.

The purpose of this talk is to determine the optimal order m to assure the boundedness of bilinear pseudo-differential operators with symbols in $BS^m_{0,0}$ from $L^p \times L^q$ to L^r in the full range $0 < p,q,r \le \infty$, where $1/p + 1/q = 1/r$ and we replace L^p by the local Hardy space h^p if $p \le 1$ and by the bmo space if $p = \infty$ (and similarly, L^q, L^r).