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Hadamard variational formula for eigenvalues of the Stokes equations and its application

Hideo KOZONO

Department of Mathematics, Waseda University, Tokyo, Japan

Abstract

Let Ω be a bounded domain in $\mathbb{R}^3$ with the smooth boundary $\partial\Omega$. Let $\rho \in C^\infty(\partial\Omega)$. For $\varepsilon > 0$, we denote by Ω_ε the domain with the boundary $\partial\Omega_\varepsilon = \{x + \varepsilon\rho(x)\nu_x ; x \in \partial\Omega\}$, where ν_x is the unit outer normal to $\partial\Omega$ at $x \in \partial\Omega$. Consider the eigenvalue problem of the Stokes equations in Ω_ε.

$$(\mathrm{S})_\varepsilon \qquad \begin{cases} -\Delta u + \nabla p = \lambda(\varepsilon) u & \text{in } \Omega_\varepsilon, \\ \operatorname{div} u = 0 & \text{in } \Omega_\varepsilon, \\ u = 0 & \text{on } \partial\Omega_\varepsilon. \end{cases}$$

We denote by $0 < \lambda_1(\varepsilon) \le \lambda_2(\varepsilon) \le \cdots\cdots$ the eigenvalue of $(\mathrm{S})_\varepsilon$ counting multiplicity. It is shown by Jimbo-Ushikoshi [1] that for every $\rho \in C^\infty(\partial\Omega)$ there exists

$$\delta\lambda_k = \lim_{\varepsilon \to +0} \frac{\lambda_k(\varepsilon) - \lambda_k(0)}{\varepsilon}, \quad k = 1, 2, \cdots .$$

Suppose that for every $k = 1, 2, \cdots$ the eigenvalue $\lambda_k(0)$ of $(\mathrm{S})_0$ has the multiplicity n_k. Then we have

Theorem. (i) *There exists $\rho \in C^\infty(\partial\Omega)$ such that for every $k = 1, 2, \cdots$ there is $k \le l \le k + n_k - 1$ such that $\delta\lambda_l \ne 0$.*

(ii) *Let $\partial\Omega$ be a closed surface in $\mathbb{R}^3$. Assume that ρ satisfies $vol.\Omega_\varepsilon = vol.\Omega$ for all $\varepsilon > 0$. If there exists $k \in \mathbb{N}$ such that*

$$\delta\lambda_l = 0 \quad \textit{for all } k \le l \le k + n_k - 1,$$

then $\partial\Omega$ is diffeomorphic to the torus in $\mathbb{R}^3$.

Miyakawa [3] obtained a similar result to the above (ii) under the assumption that $\lambda_k(0)$ is a simle eigenvalue of $(\mathrm{S})_0$. This result is based on the joint work with Profs. Shuich Jimbo, Yoshiaki Teramoto, Erika Ushikoshi [2].

References

[1] Jimbo, S., Ushikoshi, E., *Hadamard variational formula for the eigenvalue of the Stokes equations with the Dirichlet boundary conditions.*, to appear in Far East J. Math. Sci..

[2] Jimbo, S., Kozono, H., Teramoto, Y., Ushikoshi, E., *Hadamard variational formula for eigenvalues of the Stokes equations and its application.* preprint

[3] Miyakawa, T., A variational formula for the Green function of the Stokes equations. unpublished.