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Sobolev's inequality for Riesz potentials of functions in central Herz-Morrey-Orlicz spaces on the unit ball

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Abstract

The space introduced by Morrey in 1938 has become a useful tool for studying the existence and regularity of partial differential equations. Our aim in this talk is to establish Sobolev's inequality for central Herz-Morrey-Orlicz spaces on the unit ball.

We know Sobolev's inequality :

$$\|I_\alpha f\|_{L^{p_\nu,\nu}(G)} \le C\|f\|_{L^{p,\nu}(G)}$$

for $f \in M^{p,\nu}(\mathbf{R}^N)$, $0 < \alpha < \nu \le N$ and $1 < p < \nu/\alpha$, where I_α is the Riesz kernel of order α and $1/p_\nu = 1/p - \alpha/\nu \ \big(< 1/p - \alpha/N = 1/p^* \ \big)$.

Now we consider a weighted condition on f such as

$$\int_0^2 \left\{\omega(r)\|f\|_{L^p(A(0,r))}\right\}^q \frac{dr}{r} < \infty \qquad \text{when } 0 < q < \infty,$$

$$\sup_{0<r<1} \omega(r)\|f\|_{L^p(A(0,r))} < \infty \qquad \text{when } q = \infty,$$

where ω is a doubling weight, $1 < p < \infty$ and $A(0,r) = B(0,r) \setminus B(0,r/2)$ is the annulus. If this is the case, then we write $f \in H^{p,\omega,q}(\mathbf{B})$ which is called a central Herz-Morrey space on the unit ball $\mathbf{B}$. Note here that $H^{p,\omega,p}$ is a weighted L^p space.

To establish a general result, in the Orlicz settings, we give Sobolev's inequality for Riesz potentials of functions f in central Herz-Morrey-Orlicz spaces $H^{\Phi,\omega,q}(\mathbf{B})$.

In the borderline case $\alpha p = N$, we study the L^p integrability result, instead of Trudinger's inequality. Among them, we give

$$\int_{\mathbf{B}} \left\{|x|^{-\alpha}(\log(e/|x|))^{-1+\theta}|I_\alpha f(x)|\right\}^p dx$$
$$\le \quad C\int_{\mathbf{B}} \left\{(\log(e/|y|))^{\theta}|f(y)|\right\}^p dy$$

for $\theta < 1/p' = 1 - 1/p$, as in the paper by Edumunds and Triebel, 1999.