# $B_w^u$-function spaces and their interpolation[1]

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Let $\mathbb{R}^n$ be the $n$-dimensional Euclidean space. We denote by $Q_r$ the open cube centered at the origin and sidelength $2r$, or the open ball centered at the origin and of radius $r$.

**Definition 1.** *Let $w : (0,\infty) \to (0,\infty)$ be a weight function and let $u \in (0,\infty]$. We define function spaces $B_w^u(E) = B_w^u(E)(\mathbb{R}^n)$ and $\dot{B}_w^u = \dot{B}_w^u(E)(\mathbb{R}^n)$ as the sets of all functions $f \in E_Q(\mathbb{R}^n)$ such that $\|f\|_{B_w^u(E)} < \infty$ and $\|f\|_{\dot{B}_w^u(E)} < \infty$, respectively, where*

$$\|f\|_{B_w^u(E)} = \| w(r)\|f\|_{E(Q_r)}\|_{L^u([1,\infty),dr/r)},$$
$$\|f\|_{\dot{B}_w^u(E)} = \| w(r)\|f\|_{E(Q_r)}\|_{L^u((0,\infty),dr/r)}.$$

*In the above we abbreviated $\|f|_{Q_r}\|_{E(Q_r)}$ to $\|f\|_{E(Q_r)}$.*

If $E = L^p$, then $\dot{B}_w^u(L^p)(\mathbb{R}^n)$ is the local Morrey-type space introduced by Burenkov and Guliyev (Studia Math. vol. 163, 2004).

If $w(r) = r^{-\sigma}$, $\sigma \ge 0$ and $u = \infty$, we denote $B_w^u(E)(\mathbb{R}^n)$ and $\dot{B}_w^u(E)(\mathbb{R}^n)$ by $B_\sigma(E)(\mathbb{R}^n)$ and $\dot{B}_\sigma(E)(\mathbb{R}^n)$, respectively, which were introduced recently by Komori-Furuya, Matsuoka, Nakai and Sawano (Rev. Mat. Complut.vol. 26, 2013).

In this talk, we treat the interpolation property of $B_w^u$-function spaces.

**Theorem 2.** *Assume that a family $\{(E(Q_r), \|\cdot\|_{E(Q_r)})\}_{0<r<\infty}$ has the restriction and decomposition properties above. Let $u_0, u_1, u \in (0,\infty]$, $w_0, w_1 \in \mathcal{W}^\infty$, and $w = w_0^{1-\theta} w_1^{\theta}$. Assume also that, for some positive constant $\epsilon$, $(w_0(r)/w_1(r))r^{-\epsilon}$ is almost increasing, or, $(w_1(r)/w_0(r))r^{-\epsilon}$ is almost increasing. Then*

$$(\dot{B}_{w_0}^{u_0}(E)(\mathbb{R}^n), \dot{B}_{w_1}^{u_1}(E)(\mathbb{R}^n))_{\theta,u} = \dot{B}_w^u(E)(\mathbb{R}^n)$$
$$(B_{w_0}^{u_0}(E)(\mathbb{R}^n), B_{w_1}^{u_1}(E)(\mathbb{R}^n))_{\theta,u,[1,\infty)} = B_w^u(E)(\mathbb{R}^n).$$

As applications of the interpolation property, we also give the boundedness of linear and sublinear operators. It is known that the Hardy-Littlewood maximal operator, fractional maximal operators, singular and fractional integral operators are bounded on $B_\sigma$-Morrey-Campanato spaces. Interpolate these function spaces, the we get the boundedness of these operators on $B_w^u(L_{p,\lambda})$, $\dot{B}_w^u(L_{p,\lambda})$, $B_w^u(\mathcal{L}_{p,\lambda})$ and $\dot{B}_w^u(\mathcal{L}_{p,\lambda})$, which are also generalization of the results on the local Morrey-type spaces $LM_{pu,w}(\mathbb{R}^n)$.

[1]The main part of this talk is the joint work with Professor E. Nakai, Ibaraki University.