Harmonic Analysis and its Applications in Tokyo 2015
http://enakai.sci.ibaraki.ac.jp/haat2015/

On a condition for the global L^p-boundedness of Fourier integral operators

Mitsuru Sugimoto
Nagoya University, Nagoya, Japan
email: sugimoto@math.nagoya-u.ac.jp

Abstract We discuss the global $L^p(\mathbb{R}^n)$-boundedness of Fourier integral operators of the form

$$\mathcal{P}u(x) = \int_{\mathbb{R}^n}\int_{\mathbb{R}^n} e^{i\phi(x,y,\xi)} a(x,y,\xi)u(y)\,dyd\xi \quad (x \in \mathbb{R}^n), \tag{1}$$
$$\phi(x,y,\xi) = (x-y)\cdot\xi + \Phi(x,y,\xi),$$

where $\Phi(x,y,\xi)$ is introduced just for convenience and we do not lose any generality with this notation. A condition for the global $L^2(\mathbb{R}^n)$-boundedness was given by Asada and Fujiwara [1] or by the authors [2], while the local $L^p(\mathbb{R}^n)$-boundedness was given by Seeger, Sogge and Stein [3]. Unfortunately, not so many results are available for the global boundedness on $L^p(\mathbb{R}^n)$, but in this talk, we describe a construction that is used to deduce the global result from them.

Let S^k be Hörmander's symbol class and let $\mathcal{P}^*$ be the formal adjoint of $\mathcal{P}$. Then we have

Theorem 1. *Assume the following conditions:*

(A1) $\Phi(x,y,\xi)$ *is a real-valued* C^∞*-function and* $\partial_\xi^\gamma\Phi(x,y,\xi) \in S^0$ *for* $|\gamma| = 1$.
(A2) $\mathcal{P}$ *and* $\mathcal{P}^*$ *are* $L^2(\mathbb{R}^n)$*-bounded if* $a(x,y,\xi) \in S^0$.
(A3) $\mathcal{P}$ *and* $\mathcal{P}^*$ *are uniformly* $H^1_{comp}(\mathbb{R}^n)$-$L^1_{loc}(\mathbb{R}^n)$*-bounded if* $a(x,y,\xi) \in S^{-(n-1)/2}$.

Then $\mathcal{P}$ *is* $L^p(\mathbb{R}^n)$*-bounded if* $1 < p < \infty$, $\kappa \le -(n-1)|1/p - 1/2|$, *and* $a(x,y,\xi) \in S^\kappa$.

We simply state a corollary by restricting our phase functions to the form

$$\phi(x,y,\xi) = x\cdot\xi - \varphi(y,\xi) \quad (\text{in other words } \Phi(x,y,\xi) = y\cdot\xi - \varphi(y,\xi)).$$

Theorem 2. *Let* $\varphi(y,\xi)$ *and* $a(x,y,\xi)$ *be* C^∞*-functions. Assume that* $\varphi(y,\xi)$ *is positively homogeneous of order* 1 *for large* ξ *and satisfies*

$$|\det \partial_y\partial_\xi\varphi(y,\xi)| \ge C > 0,$$

Also assume that

$$\left|\partial_y^\alpha\partial_\xi^\beta(y\cdot\xi - \varphi(y,\xi))\right| \le C_{\alpha\beta}\langle\xi\rangle^{1-|\beta|} \quad (\forall\,\alpha, |\beta| \ge 1),$$
$$\left|\partial_x^\alpha\partial_y^\beta\partial_\xi^\gamma a(x,y,\xi)\right| \le C_{\alpha\beta\gamma}\langle\xi\rangle^{-(n-1)|1/p-1/2|-|\gamma|} \quad (\forall\alpha,\beta,\gamma),$$

hold for all $x, y, \xi \in \mathbb{R}^n$. *Then* $\mathcal{P}$ *is* $L^p(\mathbb{R}^n)$*-bounded for every* $1 < p < \infty$.

This is joint work with Michael Ruzhansky (Imperial College London)

Bibliography

[1] K. Asada and D. Fujiwara. On some oscillatory integral transformations in $L^2(\mathbf{R}^n)$. *Japan. J. Math. (N.S.)*, 4(2):299–361, 1978.
[2] M. Ruzhansky and M. Sugimoto. Global L^2-boundedness theorems for a class of Fourier integral operators. *Comm. Partial Differential Equations*, 31(4-6):547–569, 2006.
[3] A. Seeger, C. D. Sogge, and E. M. Stein. Regularity properties of Fourier integral operators. *Ann. of Math. (2)*, 134(2):231–251, 1991.