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ON THE BOUNDEDNESS OF MULTILINEAR FOURIER MULTIPLIER OPERATORS

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For $m(\xi,\eta,\sigma) \in L^\infty(\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n)$, the trilinear Fourier multiplier operator T_m is defined by

$$T_m(f,g,h)(x) = \int_{\mathbb{R}^{3n}} e^{ix\cdot(\xi+\eta+\sigma)} m(\xi,\eta,\sigma)\widehat{f}(\xi)\widehat{g}(\eta)\widehat{h}(\sigma)\,d\xi d\eta d\sigma,$$

where $x \in \mathbb{R}^n$, f,g,h are the Schwartz functions on $\mathbb{R}^n$ and $\widehat{f},\widehat{g},\widehat{h}$ are their Fourier transforms. We say that a function m on $\mathbb{R}^d \setminus \{0\}$ belongs to the class $\mathcal{M}(\mathbb{R}^d)$ if

$$|\partial^\alpha m(\zeta)| \le C_\alpha |\zeta|^{-|\alpha|}, \qquad \zeta \in \mathbb{R}^d \setminus \{0\},$$

for each α. As a typical example of trilinear flag paraproducts, Muscalu [3] considered a multiplier of the form

$$m(\zeta) = m^{II}(\xi,\eta)\widetilde{m}^{II}(\eta,\sigma), \qquad \zeta = (\xi,\eta,\sigma) \in (\mathbb{R}^n)^3$$

where $m^{II}, \widetilde{m}^{II} \in \mathcal{M}(\mathbb{R}^{2n})$. On the other hand, in problems of partial differential equations, Germain-Masmoudi-Shatah [1, 2] considered a multiplier of the form

$$m(\zeta) = m^{III}(\xi,\eta,\sigma)m^{II}(\eta,\sigma)\widetilde{m}^{II}(\xi,\eta+\sigma), \qquad \zeta = (\xi,\eta,\sigma) \in (\mathbb{R}^n)^3,$$

where $m^{III} \in \mathcal{M}(\mathbb{R}^{3n})$ and $m^{II}, \widetilde{m}^{II} \in \mathcal{M}(\mathbb{R}^{2n})$. In [1, 2, 3], the boundedness of $L^p \times L^q \times L^r$ to L^s, $p,q,r > 1$, $1/p + 1/q + 1/r = 1/s$, was discussed. The purpose of this talk is to consider the boundedness of trilinear flag paraproducts in the full range $p,q,r > 0$ by using Hardy spaces.

We also introduce the recent result on multilinear Fourier multipliers with minimal Sobolev regularity.

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References

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[3] C. Muscalu, Paraproducts with flag singularities I. A case study, Rev. Mat. Iberoam. 23 (2007), 705–742.