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Remarks on the strong maximum principle involving p-Laplacian

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Abstract

Let $N \geq 1$, $1 < p < \infty$ and $p^* = \max(1, p-1)$. Let Ω be a bounded domain of $\mathbf{R}^N$. We shall study the strong maximum principle for the operator; $-\Delta_p + a(x)Q(\cdot)$. Here $a \geq 0$ a.e. in Ω, Δ_p is a p-Laplacian and $Q(\cdot)$ is a nonlinear term. Let $u \in L^1_{\text{loc}}(\Omega)$ if $p = 2$ and let $u \in W^{1,p^*}_{\text{loc}}(\Omega)$ if $p \neq 2$. We assume that $u \geq 0$ a.e. in Ω, $Q(u) \in L^1_{\text{loc}}(\Omega)$ and $\Delta_p u$ is a Radon measure on Ω. When $p \neq 2$, we also assume that u is **admissible**. Moreover, we assume that $-\Delta_p u + a(x)Q(u) \geq 0$ in Ω in the measure sense. Then we prove that if $\tilde{u} = 0$ on a set of positive $p-$capacity in Ω, then $u = 0$ a.e. in Ω. Here $\tilde{u}$ is a quasicontinuous representative of u.