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Dispersive estimates for the stably stratified Boussinesq equations

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We consider the initial value problem for the 3D Boussinesq equations for stably stratified fluids without the rotational effect.

$$\begin{cases} \partial_t u + (u \cdot \nabla) u = \Delta u - \nabla p + \theta e_3 & t > 0,\ x \in \mathbb{R}^3, \\ \partial_t \theta + (u \cdot \nabla)\theta = \Delta \theta - N^2 u_3 & t > 0,\ x \in \mathbb{R}^3, \\ \nabla \cdot u = 0 & t > 0,\ x \in \mathbb{R}^3, \\ u(0,x) = u_0(x), \quad \theta(0,x) = \theta_0(x) & x \in \mathbb{R}^3. \end{cases} \qquad (B_N)$$

Here, $u = (u_1(t,x), u_2(t,x), u_3(t,x))^T$, $p = p(t,x)$ and $\theta = \theta(t,x)$ are the unknown functions, representing the velocity field, the scalar pressure and the thermal disturbance about a mean state in hydrostatic balance, respectively, while $u_0 = (u_{0,1}(x), u_{0,2}(x), u_{0,3}(x))^T$ is the given initial velocity field and $\theta_0 = \theta_0(x)$ is the given initial thermal disturbance. $N > 0$ is the Brunt-Väisälä (buoyancy) frequency for the constant stratification. We establish the sharp dispersive estimate for the linear propagator related to the stable stratification. As an application, we give the explicit relation between the size of initial data and the buoyancy frequency which ensures the unique existence of global solutions to our system (B_N). In particular, it is shown that the size of the initial thermal disturbance can be taken in proportion to the strength of stratification.