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Variable Hardy-Lorentz spaces $H^{p(\cdot),q}$

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Let $p(\cdot)$ be a measurable function on $\mathbb{R}^n$ with $0 < p_- := \operatorname{ess\,inf}_{x\in\mathbb{R}^n} p(x) \leq \operatorname{ess\,sup}_{x\in\mathbb{R}^n} p(x) =: p_+ < \infty$. In this talk, we introduce the variable Hardy-Lorentz space $H^{p(\cdot),q}(\mathbb{R}^n)$ for $0 < q \leq \infty$ via the radial grand maximal function. Under the assumption that $p(\cdot)$ satisfies the log-Hölder condition,we establish a version of Fefferman-Stein vector-valued inequality in variable Lorentz space $L^{p(\cdot),q}(\mathbb{R}^n)$ by interpolation. We also construct atomic decompositions for $H^{p(\cdot),q}(\mathbb{R}^n)$, and develop a theory of real interpolation and formulate the dual space of the variable Hardy-Lorentz space $H^{p(\cdot),q}(\mathbb{R}^n)$ with $0 < p_- \leq p_+ \leq 1$ and $0 < q < \infty$. As a byproduct, we obtain a new John-Nirenberg theorem. If possible, I also would like to talk about some similar results in the martingale setting.