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RECENT DEVELOPMENTS OF THE WEIGHT THEORY ON MORREY SPACES

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In this talk based on the joint works with Professors Takeshi Iida, Yoshihiro Sawano and Hitoshi Tanaka, we give the recent developments of the weight theory on Morrey spaces established in [1, 2, 3, 4]. There exists two way to define weighted Morrey spaces, the one is due to Komori-Shirai and another is due to Samko;

$$\|f\|_{\mathcal{M}^p_q(w)} = \sup_{Q:\text{cube}} |Q|^{\frac{1}{p}-\frac{1}{q}} \left(\int_Q |f(x)|^q w(x) dx \right)^{\frac{1}{q}}.$$

We are interested in Samko-type weighted Morrey spaces.

Our talk is constructed from three parts;

(1) The Fefferman-Stein type inequality on $\mathcal{M}^p_q(w)$. Recall that the F-S type inequality holds for any weight w in the Lebesgue setting. In contrast, it turns out that the F-S type inequality does not always hold in the Morrey setting.
(2) The boundedness of the Riesz transform R_i on $\mathcal{M}^p_q(w)$ and the Morrey counterpart of the A_p condition. Our result is not enough to characterize the weighted inequality for the Hardy-Littlewood maximal operator completely. So, the problem regarding to the H-L maximal operator is still open.
(3) The boundedness of the fractional integral operator I_α on $\mathcal{M}^p_q(w)$ and the Morrey counterpart of the $A_{p,q}$ condition. Again, our result is not enough to settle down the problem regarding to the maximal operator.

Key conditions imposed on weight w we introduced are the following:

- w satisfies that for all cube Q,
$$\|\chi_Q\|_{\mathcal{M}^p_q(w)} \sim \Phi_{p,q,w}(Q) := |Q|^{\frac{1}{p}-\frac{1}{q}} w(Q)^{\frac{1}{q}}.$$
- w satisfies that for all cube Q,
$$\int_1^\infty \frac{1}{\Phi_{p,q,w}(sQ)} \frac{ds}{s} \le \frac{C}{\Phi_{p,q,w}(Q)}.$$

References

[1] T. Iida and S. Nakamura, A note on Fefferman-Stein inequality on Morrey spaces, Rev. Mat. Complut., (2016) DOI 10.1007/s13163-016-0217-y.
[2] S. Nakamura, Generalized weighted Morrey spaces and classical operators, Math. Nachr., (2016) DOI: 10.1002/mana.201500260.
[3] S. Nakamura, Y. Sawano, *The singular integral operator and its commutator on weighted Morrey spaces*, Coll. Math. **68** (2017), no. 2, 145–174.
[4] S. Nakamura, Y. Sawano, H. Tanaka, *Fractional operators on weighted Morrey spaces*, in arXiv.