The Carleson type embedding inequality for dyadic rectangles

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In this talk we introduce the Carleson type embedding inequality for dyadic rectangles and consider some of its applications.

We denote the set of all dyadic rectangles on $\mathbb{R}^n$ by

$$D_R(\mathbb{R}^n) = \{2^{-k}(m + [0, 1)) : k, m \in \mathbb{Z} \}^n.$$

We denote by $P_i$, $i = 1, 2, \ldots, n$, the projection on the $x_i$-axis. For $R \in D_R(\mathbb{R}^n)$, $I \in D(\mathbb{R})$ and $j = 1, 2, \ldots, n$, we define the dyadic rectangle

$$R_{I,j} = \left( \prod_{i=1}^{j-1} P_i(R) \right) \times I \times \left( \prod_{i=j+1}^n P_i(R) \right).$$

**Theorem** Given a weight $\sigma$ in $\mathbb{R}^n$ and $1 < p < q < \infty$, the following statements are equivalent:

(a) The Carleson type embedding inequality for rectangles

$$\sum_{R \in D_R(\mathbb{R}^n)} \sigma(R)^{q/p} \left( \int_R f \, d\sigma \right)^q \leq c_1 \left( \int_{\mathbb{R}^n} f^p \, d\sigma \right)^{q/p}$$

holds for all nonnegative function $f \in L^p(\sigma)$;

(b) The testing condition

$$\sum_{I \subset P_j(R)} \sigma(R_{I,j})^{q/p} \leq c_2 \sigma(R)^{q/p}$$

holds for all $R \in D_R(\mathbb{R}^n)$ and $j = 1, 2, \ldots, n$.

Moreover, the least possible constants $c_1$ and $c_2$ enjoy $c_1 \leq Cc_2^n$ and $c_2 \leq c_1$.

In this talk we introduce the proof of this theorem, which is due to Professor Yabuta, and give its some applications.

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