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The Carleson type embedding inequality for dyadic rectangles

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In this talk we introduce the Carleson type embedding inequality for dyadic rectangles and consider some its applications.

We denote the set of all dyadic rectangles on $\mathbb{R}^n$ by

$$\mathcal{D}R(\mathbb{R}^n)=\{2^{-k}(m+[0,1)):\,k,m\in\mathbb{Z}\}^n.$$

We denote by P_i, $i=1,2,\ldots,n$, the projection on the x_i-axis. For $R\in\mathcal{D}R(\mathbb{R}^n)$, $I\in\mathcal{D}(\mathbb{R})$ and $j=1,2,\ldots,n$, we define the dyadic rectangle

$$R_{I,j}=\left(\prod_{i=1}^{j-1}P_i(R)\right)\times I\times\left(\prod_{i=j+1}^{n}P_i(R)\right).$$

Theorem Given a weight σ in $\mathbb{R}^n$ and $1<p<q<\infty$, the following statements are equivalent:

(a) The Carleson type embedding inequality for rectangles

$$\sum_{R\in\mathcal{D}R(\mathbb{R}^n)}\sigma(R)^{q/p}\left(\fint_R f\,d\sigma\right)^q\le c_1\left(\int_{\mathbb{R}^n}f^p\,d\sigma\right)^{q/p}$$

holds for all nonnegative function $f\in L^p(\sigma)$;

(b) The testing condition

$$\sum_{\substack{I\in\mathcal{D}(\mathbb{R})\\ I\subset P_j(R)}}\sigma(R_{I,j})^{q/p}\le c_2\sigma(R)^{q/p}$$

holds for all $R\in\mathcal{D}R(\mathbb{R}^n)$ and $j=1,2,\ldots,n$.

Moreover, the least possible constants c_1 and c_2 enjoy $c_1\le Cc_2^n$ and $c_2\le c_1$.

In this talk we introduce the proof of this theorem, which is due to Professor Yabuta, and give its some applications.

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