Bilinear pseudo-differential operators of type 1,1 and their application to the Kato-Ponce inequality

Naohito Tomita (Osaka University)

The Hörmander class $S^m_{\rho,\delta}$ $m \in \mathbb{R}$, $0 \le \delta \le \rho \le 1$, consists of all $\sigma(x,\xi) \in C^\infty(\mathbb{R}^n \times \mathbb{R}^n)$ such that

$$|\partial_x^\alpha \partial_\xi^\beta \sigma(x,\xi)| \le C_{\alpha,\beta}(1+|\xi|)^{m+\delta|\alpha|-\rho|\beta|}$$

for all multi-indices α, β. For $\sigma \in S^m_{\rho,\delta}$, the pseudo-differential operator $\sigma(X,D)$ is defined by

$$\sigma(X,D)f(x) = \frac{1}{(2\pi)^n}\int_{\mathbb{R}^n} e^{ix\cdot\xi}\sigma(x,\xi)\widehat{f}(\xi)\,d\xi,$$

where f is a Schwartz function on $\mathbb{R}^n$ and $\widehat{f}$ is the Fourier transform of f. It is known that if $\delta < 1$, then all pseudo-differential operators with symbols in $S^0_{\rho,\delta}$ are bounded on L^2. However, in the case $\delta = 1$, there is a symbol $\sigma(x,\xi) \in S^0_{1,1}$ such that $\sigma(X,D)$ is not bounded on L^2. This means a gap between the cases $\delta < 1$ and $\delta = 1$.

In this talk, we consider pseudo-differential operators of type 1,1 (namely, $\rho = \delta = 1$) in the bilinear setting. As an application, the Kato-Ponce type inequality is discussed in local Hardy spaces.