

## Boundedness and compactness of Riesz transform commutator in the Bessel setting

Dongyong Yang

School of Mathematical Sciences, Xiamen University

**Abstract:** Let  $\lambda \in (0, \infty)$ ,  $p \in (1, \infty)$  and  $\Delta_\lambda = -\frac{d^2}{dx^2} - \frac{2\lambda}{x} \frac{d}{dx}$  be the Bessel operator on  $\mathbb{R}_+ = (0, \infty)$ . We show that a function  $b$  is in the BMO space  $\text{BMO}(\mathbb{R}_+, x^{2\lambda} dx)$  if and only if the commutator  $[b, R_{\Delta_\lambda}]$  of the Riesz transform  $R_{\Delta_\lambda}$  is bounded on  $L^p(\mathbb{R}_+, x^{2\lambda} dx)$ . Moreover, we also show that a function  $b$  is in  $\text{CMO}(\mathbb{R}_+, x^{2\lambda} dx)$  if and only if  $[b, R_{\Delta_\lambda}]$  is compact on  $L^p(\mathbb{R}_+, x^{2\lambda} dx)$ . These results are based on the joint work with Xuan Thinh Duong, Ji Li, Suzhen Mao, Brett D. Wick and Huoxiong Wu.