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On Selberg's integral formula

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In this talk, we investigate some necessary and sufficient conditions which ensure validity of the Selberg's integral formula. That is, the Selberg's integral equation

$$\int_{\mathbb{R}^n} \prod_{i=1}^{k} |x^i - t|^{-d_i} dt = C_{d_1,\cdots,d_k,n} \prod_{1\le i<j\le k} |x^i - x^j|^{-\alpha_{ij}}$$

holds for any $x^i \in \mathbb{R}^n$ and some nonzero real numbers d_i with $i = 1, \cdots, k$ if and only if one of the following two conditions holds.

Condition I is that $k = 2$ and $\max\{d_1, d_2\} < n < d_1 + d_2$;

Condition II is that $k = 3$, $\max\{d_1, d_2, d_3\} < n$ and $d_1 + d_2 + d_3 = 2n$.

Actually, we completely answer one question raised by Grafakos in the reference [**?**].

In fact, for some cases, the constant number $C_{d_1,\cdots,d_k,n}$ is just the sharp bound of the following Hardy-Littlewood -Sobolev inequality

$$\left| \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} \frac{f(x)g(y)}{|x|^{\alpha}|x-y|^{\lambda}|y|^{\beta}} dxdy \right| \le C(p, q, \alpha, \lambda, \beta, n) \|f\|_{L^p(\mathbb{R}^n)} \|g\|_{L^q(\mathbb{R}^n)}.$$